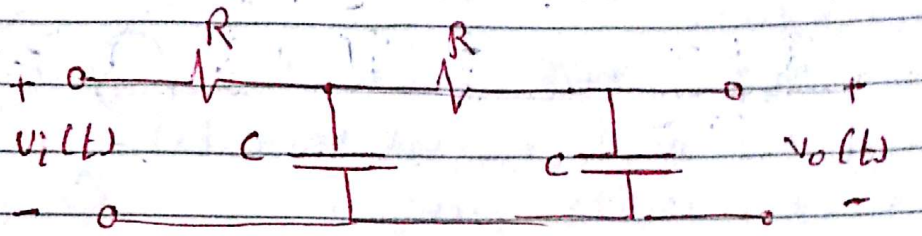
Modeling

Page Sheet 1
Date

[1] Find the T.F. of the following electrical systems: (2 درجات)

(a)



prove $T.F. = \frac{V_o(s)}{V_i(s)} = \frac{\frac{1}{R^2C^2}}{s^2 + \frac{3}{RC}s + \frac{1}{R^2C^2}}$

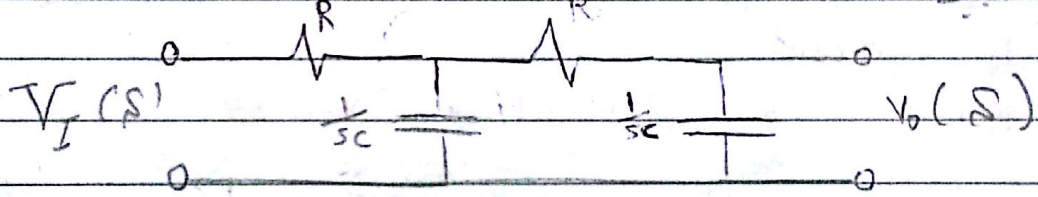
① نحول الدائرة من (T-domain) إلى (S-domain)

$R \longrightarrow R$

$L \longrightarrow sL$

$C \longrightarrow \frac{1}{sC}$

نتائج تحويل لابلاس



$$Z_{eq} = R + \left(\frac{1}{sC} \,//\, \left(R + \frac{1}{sC}\right)\right)$$

$$= R + \left(\frac{\frac{1}{sC} * \left(R + \frac{1}{sC}\right)}{\frac{1}{sC} + R + \frac{1}{sC}} * \frac{s^2C^2}{s^2C^2}\right)$$

$$= R + \frac{RsC + 1}{sC + Rs^2C^2 + sC}$$

$$= R + \frac{1 + RsC}{Rs^2C^2 + 2sC}$$

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$$Z_{eq} = \frac{R^2S^2C^2 + 2RSC + RSC + 1}{RS^2C^2 + 2SC}$$

$$Z_{eq} = \frac{R^2S^2C^2 + 3RSC + 1}{RS^2C^2 + 2SC} \quad \#\#$$

$$V_o(S) = I_2(S) \cdot \frac{1}{SC}$$

$$V_{in}(S) = I_1(S) \cdot Z_{eq}$$

$$I_2(S) = I_1(S) \cdot \frac{(1/SC)}{(R + \frac{1}{SC} + \frac{1}{SC})}$$ ⟸ استخدام قانون تقسيم التيار

$$\frac{V_o(S)}{V_i(S)} = \frac{I_1(S) \times \left(\frac{1/SC}{R + \frac{2}{SC}}\right) \times \frac{S^2C^2}{S^2C^2}}{I_1(S) \cdot \left(\frac{R^2S^2C^2 + 3RSC + 1}{RS^2C^2 + 2SC}\right)}$$

$$= \frac{\frac{1}{RS^2C^2 + 2SC}}{\frac{R^2S^2C^2 + 3RSC + 1}{RS^2C^2 + 2SC}} = \frac{1}{R^2S^2C^2 + 3RCS + 1}$$

نقسم البسط والمقام على R^2C^2

$$\frac{V_o(S)}{V_i(S)} = \frac{\frac{1}{R^2C^2}}{S^2 + \frac{3}{RC}S + \frac{1}{R^2C^2}} \quad \#\#$$

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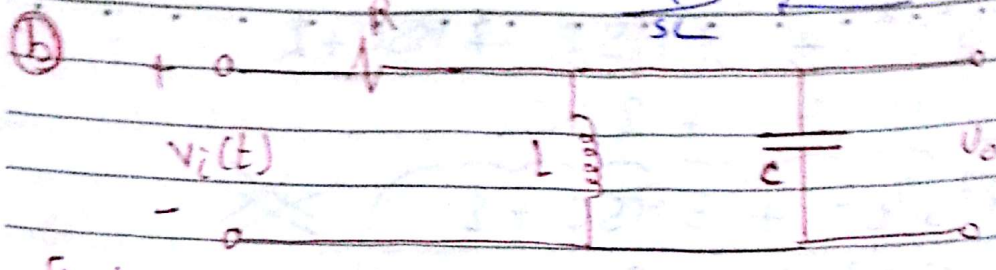
S domain ⟵ (T. domain) ⟵ ω

R ⟵ R

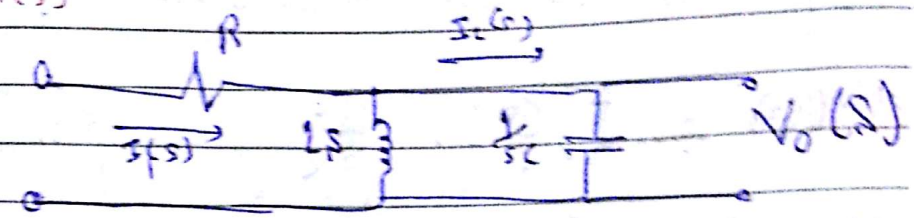
SL ⟵ jωL

$\frac{1}{SC}$ ⟵ $\frac{1}{jωC}$

(b)



Find T.F = $\frac{V_o(s)}{V_i(s)}$

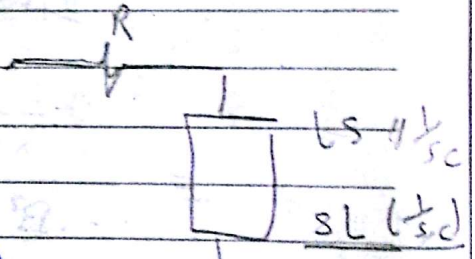


$$Z_{eq} = R + (Ls \,//\, \tfrac{1}{sC})$$

$$= R + \frac{Ls * \frac{1}{sC}}{Ls + \frac{1}{sC}} * \frac{sC}{sC}$$

$$= R + \frac{Ls}{LCs^2 + 1}$$

$$= \frac{RLCs^2 + R + Ls}{1 + s^2LC}$$



$$\frac{sL(\frac{1}{sC})}{Ls + \frac{1}{sC}} = \frac{\frac{L}{C}}{Ls + \frac{1}{sC}} = \frac{Ls}{LCs^2 + 1}$$

$$V_o(s) = I_1(s) \cdot \frac{Ls}{LCs^2 + 1}$$

$$V_i(s) = I_1(s) \cdot Z_{eq}$$

$$\therefore \frac{V_o(s)}{V_i(s)} = \frac{I_1 * \frac{Ls}{LCs^2 + 1}}{I_1(s) * \frac{RLCs^2 + Ls + R}{1 + LCs^2}}$$

$$= \frac{Ls}{RLCs^2 + Ls + R} = \frac{\frac{1}{RC}s}{s^2 + \frac{1}{RC}s + \frac{1}{LC}}$$

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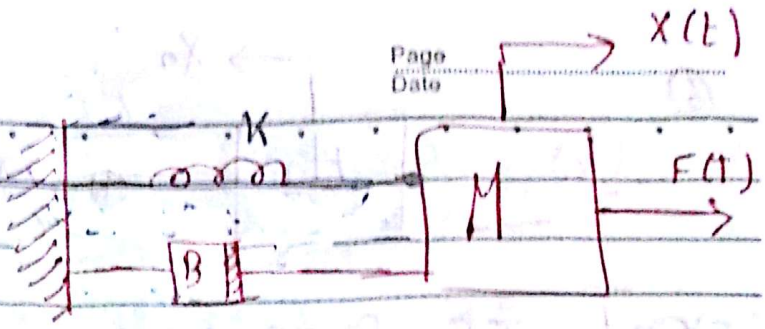
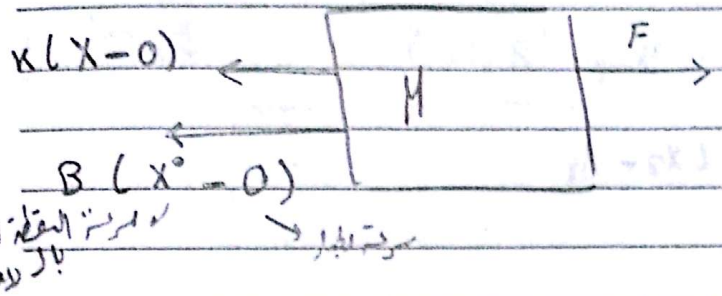
⑤ Find $\frac{X(s)}{F(s)}$



[1] نفس الـ force عند الـ external force وفيه قوة من الـ Spring وفيه قوة من الـ Damper

at M



سرعة بالبداية — سرعة البقعة الخاصة بالـ mass

$$\boxed{\Sigma F = M\ddot{X}}$$

$$M\ddot{X} = F - B\dot{X} - KX$$

$$\boxed{F(t) = M\ddot{X} + B\dot{X} + KX} \quad \text{D.E.}$$

L.T $\quad F(S) = (MS^2 + BS + K)\,X(S)$

$$\boxed{T.F = \frac{F(S)}{X(S)} = \frac{1}{MS^2 + BS + K}} \quad \# \#$$

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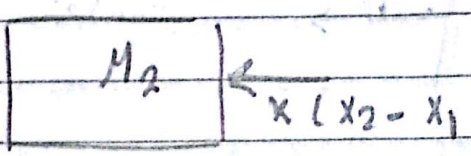
Find T.F from F to x_1 $\left(\frac{X_1(s)}{F(s)}\right)$

At M_2



$$\Sigma F = m_2 x_2''$$

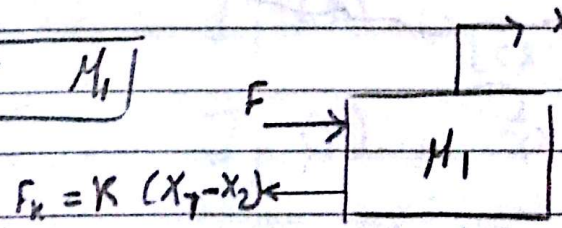
$$-K(x_2 - x_1) = m_2 x_2''$$

$$-K x_2 + K x_1 = M_2 x_2''$$ نرتب حسب x_2

$$\Rightarrow M x_2'' + K x_2 = K x_1$$

L.T: $$(M_2 s^2 + K) X_2(s) = K X_1(s)$$ ⟸ 1

AT M_1



$$\Sigma F = M_1 x_1''$$

$$F - K(x_1 - x_2) = M_1 x_1''$$

$$F(t) - K x_1 + K x_2 = M_1 x_1''$$

$$F(s) - K X_1(s) + K X_2(s) = M_1 s^2 X_1(s)$$

نعوض عن X_2 في المعادلة الأولى

$$F(s) + \frac{K^2}{M_2 s^2 + K} X_1(s) - K X_1(s) = M_1 s^2 X_1(s)$$

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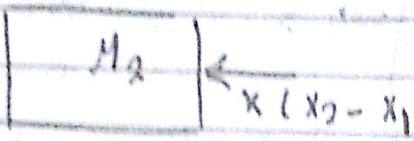
(5)

Find T.F. from F to X_1 $\left(\frac{X_1(s)}{F(s)}\right)$

At M_2



$$\Sigma F = m_2 X_2''$$

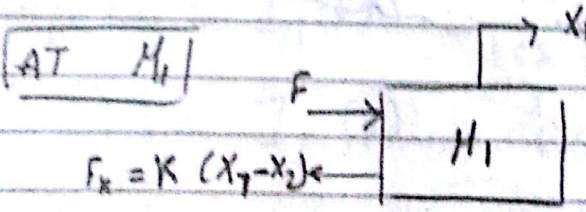
$$-K(X_2 - X_1) = m_2 X_2''$$

$$-K X_2 + K X_1 = M_2 X_2''$$ نجمع X_2 مع بعض

$$\Rightarrow M X_2'' + K X_2 = K X_1$$

L.T: $$(M_2 s^2 + K) X_2(s) = K X_1(s) \quad \Leftarrow 1$$

AT M_1



$$\Sigma F = M_1 X_1''$$

$$F - K(X_1 - X_2) = M_1 X_1''$$

$$F(t) - K X_1 + K X_2 = M_1 X_1''$$

$$F(s) - K X_1(s) + K X_2(s) = M_1 s^2 X_1(s)$$

نعوض عن X_2 الموجود بالأعلى

$$F(s) + \frac{K^2}{M_2 s^2 + K} X_1(s) - K X_1(s) = M_1 s^2 X_1(s)$$

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$$F(s) = \left(M_1 s^2 + K - \frac{K^2}{M_2 s^2 + K} \right) X_1(s)$$

$$F(s) = \left(\frac{(M_1 s^2 + K)(M_2 s^2 + K) - K^2}{M_2 s^2 + K} \right) X_1(s)$$

$$\frac{X_1(s)}{F(s)} = \frac{M_2 s^2 + K}{(M_1 s^2 + K)(M_2 s^2 + K) - K^2}$$

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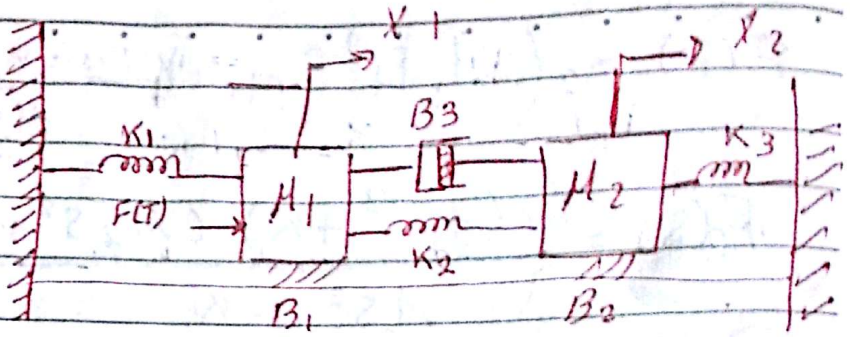
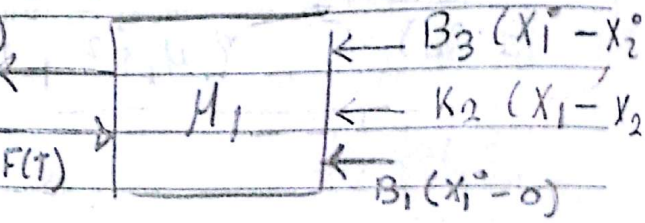
(7) Find $\frac{X_2(s)}{F(s)}$



(1) AT M_1



$\Sigma F(t) = M_1 X_1''$

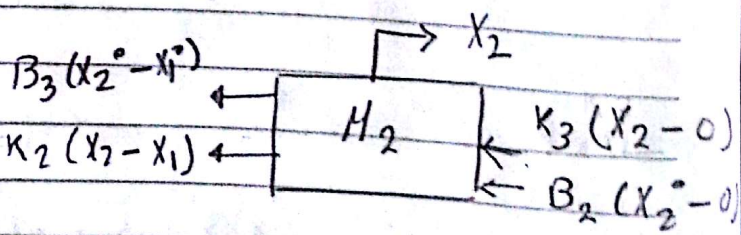
$F(t) - K_1 X_1 - B_1 X_1' - K_2 (X_1 - X_2) - B_3 (X_1' - X_2') = M_1 X_1''$

$F(t) - K_1 X_1 - B_1 X_1' - K_2 X_1 + K_2 X_2 - B_3 X_1' + B_3 X_2' = M_1 X_1''$

↳ (L.T.)

$$F(s) - (K_1 + B_1 s + K_2 + B_3 s + M_1 s^2) X_1(s) = (-K_2 - B_3 s) X_2(s) \quad (1)$$

(2) At M_2



$\Sigma F = M_2 X_2''$

$-B_3 (X_2' - X_1') - K_2 (X_2 - X_1) - K_3 X_2 - B_2 X_2' = M_2 X_2''$

$-B_3 X_2' + B_3 X_1' - K_2 X_2 + K_2 X_1 - K_3 X_2 - B_2 X_2' = M_2 X_2''$

↳ L.T. . . . . .

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$$-B_3 S X_2(s) + B_3 S X_1(s) - K_2 X_2(s) + K_2 X_1(s) - K_3 X_2(s) - B_2 S X_2(s) = M_2 S^2 X_2(s)$$

$$(M_2 S^2 + B_3 S + K_2 + K_3 + B_2 S) X_2(s) = (B_3 S + K_2) X_1(s)$$

$$X_1(s) = \frac{M_2 S^2 + (B_3 + B_2) S + K_2 + K_3}{(B_3 S + K_2)} X_2(s)$$

بالتعويض في (1) ..

$$F(s) = \frac{(M_1 S^2 + (B_1 + B_3) S + K_1 + K_2)(M_2 S^2 + (B_3 + B_2) S + K_2 + K_3) - (B_3 S + K_2)^2}{(B_3 S + K_2)} X_2(s)$$

$$X_2(s) = (-K_2 - B_3 S) X_2(s)$$

$$F(s) = \frac{(M_1 S^2 + (B_1 + B_3) S + K_1 + K_2)(M_2 S^2 + (B_3 + B_2) S + K_2 + K_3) - (B_3 S + K_2)^2}{(B_3 S + K_2)} X_2(s)$$

$$\frac{X_2(s)}{F(s)} = \frac{(B_3 S + K_2)}{(M_1 S^2 + (B_1 + B_3) S + K_1 + K_2)(M_2 S^2 + (B_3 + B_2) S + K_2 + K_3) - (B_3 S + K_2)^2}$$

$$\frac{1}{\left(\frac{M_1 S^2 + (B_1 + B_3) S + K_1 + K_2}{(B_3 S + K_2)}\right)(M_2 S^2 + (B_3 + B_2) S + K_2 + K_3) - (B_3 S + K_2)}$$

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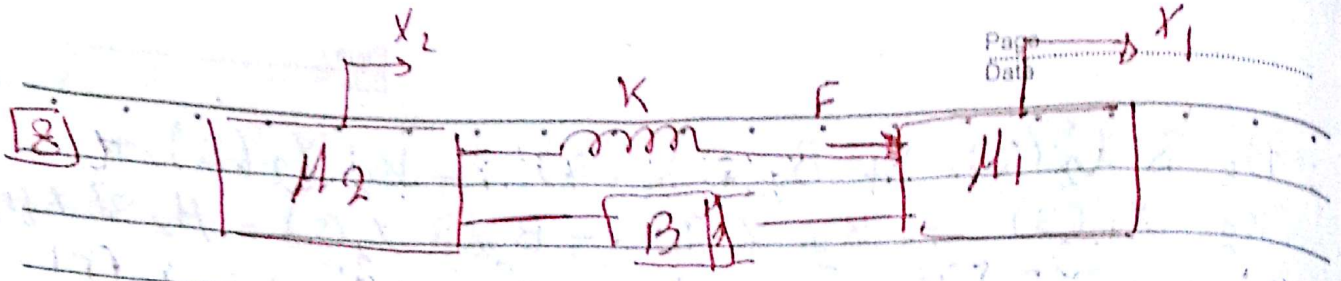
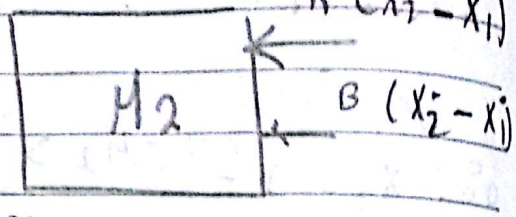


Q8)

Find T.F. $= \frac{X_1(s)}{F(s)}$

at M_2



$$\Sigma F = M_2 X_2''$$

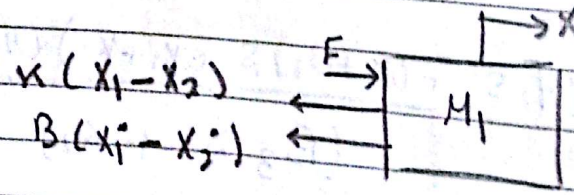
$$-K(X_2 - X_1) - B(X_2' - X_1') = M_2 X_2''$$

$$-KX_2 + KX_1 - BX_2' + BX_1' = M_2 X_2'' \quad \text{...(1)}$$

$$(BS + K) X_1(s) = (M_2 S^2 + BS + K) X_2(s)$$

$$\frac{X_2(s)}{X_1(s)} = \frac{BS + K}{M_2 S^2 + BS + K} \quad \text{...(1)}$$

at M_1



$$\Sigma F = M_1 X_1''$$

$$F(t) - K(X_1 - X_2) - B(X_1' - X_2') = M_1 X_1''$$

$$F(t) - KX_1 + KX_2 - BX_1' + BX_2' = M_1 X_1''$$

$$F(s) - KX_1(s) + KX_2(s) - BSX_1(s) + BSX_2(s) = M_1 S^2$$

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$$F(s) + (Bs + K) X_2(s) = (M_1 s^2 + Bs + K) X_1(s)$$

$$F(s) + \frac{(Bs+K)^2}{M_2 s^2 + Bs + K} X_1(s) = (M_1 s^2 + Bs + K) X_1(s)$$

$$F(s) = \left[(M_1 s^2 + Bs + K) - \frac{(Bs+K)^2}{M_2 s^2 + Bs + K} \right] X_1(s)$$

$$F(s) = \frac{(M_1 s^2 + Bs + K)(M_2 s^2 + Bs + K) - (Bs+K)^2}{M_2 s^2 + Bs + K} X_1(s)$$

$$\frac{X_1(s)}{F(s)} = \frac{M_2 s^2 + Bs + K}{(M_1 s^2 + Bs + K)(M_2 s^2 + Bs + K) - (Bs+K)^2}$$

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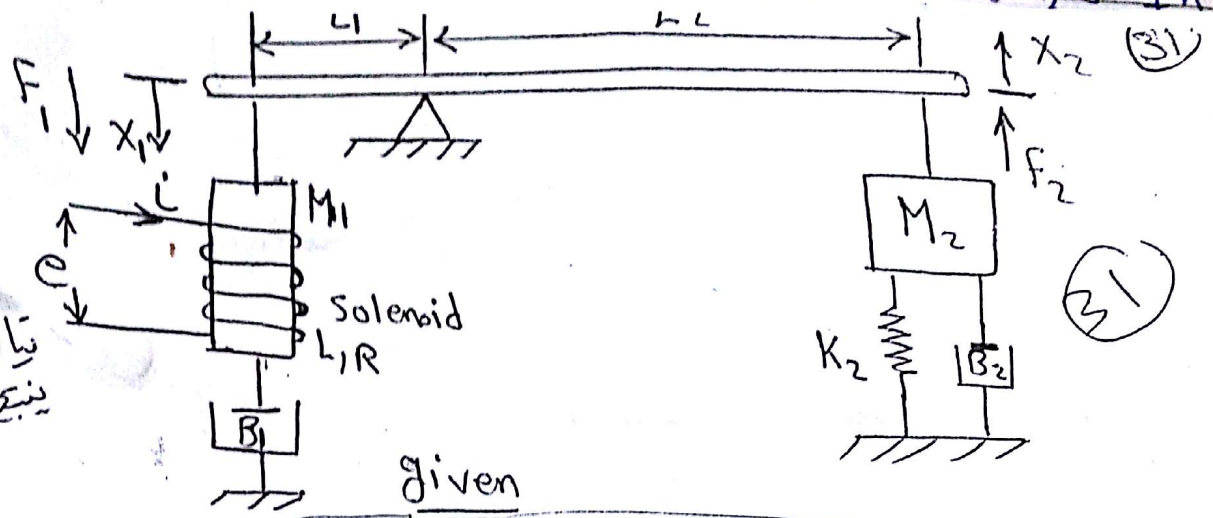
X_2

From (4) $\quad X_1(s) = \frac{-1}{(m_2 s^2 + K)}$



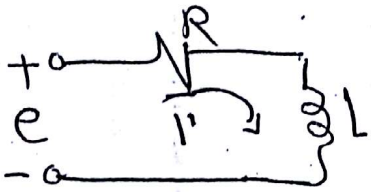
تيار يمر
ينتج مجال مغناطيسي

given

① $F_1 = Ki$ ② $\frac{X_1}{L_1} = \frac{X_2}{L_2}$ or $F_1 L_1 = F_2 L_2$

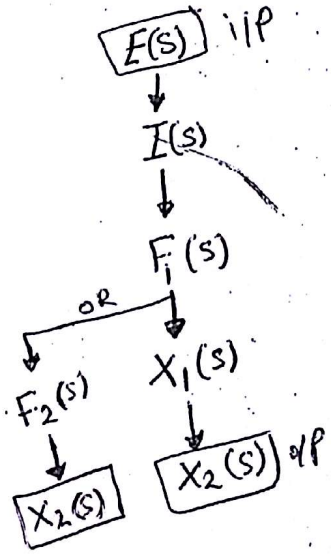
Find ① $T.F = \frac{X_2(s)}{E(s)}$?? ② T.F of each block

Q(s)

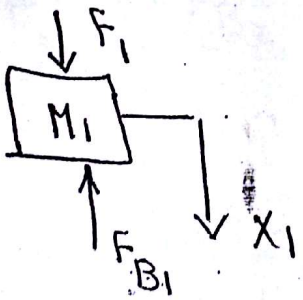


K.V.L $\Rightarrow e = Ri + L\frac{di}{dt}$ $\downarrow$ L.T

$\Rightarrow \boxed{E(s) = (R + Ls) I(s)}$ ①



at M_1



$\sum F's = M_1 \ddot{X}_1$

$F_1 - F_{B_1} = M_1 \ddot{X}_1$

$F_1 - B_1(\dot{x}_1 - 0) = M_1 \ddot{X}_1$

$F_1 = M_1 \ddot{X}_1 + B_1 \dot{X}_1 \xrightarrow{L.T} \boxed{F_1(s) = (M_1 s^2 + B_1 s) X_1(s)}$ ②

given $F_1 = Ki \Rightarrow F_1(s) = K I(s)$ ③

$\Rightarrow I(s) = \frac{1}{K} F_1(s) \Rightarrow \boxed{I(s) = \frac{1}{K}(M_1 s^2 + B_1 s) X_1(s)}$

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From ③ in ① ⇒ (32')

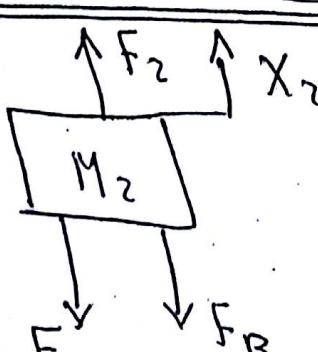
$$E(s) = (R+Ls)\frac{1}{K}(M_1 s^2 + B_1 s) X_1(s) \quad (4)$$

$$\therefore \frac{X_1}{L_1} = \frac{X_2}{L_2} \Rightarrow X_1(s) = \frac{L_1}{L_2} X_2(s) \quad (5)$$

From ⑤ in ④ ⇒ $E(s) = \frac{1}{K}(R+Ls)(M_1 s^2 + B_1 s)\left(\frac{L_1}{L_2}\right) X_2(s)$

$$T.F = \frac{X_2(s)}{E(s)} = \frac{K\left(\frac{L_2}{L_1}\right)}{(R+Ls)(M_1 s^2 + B_1 s)} \quad *$$

at M_2

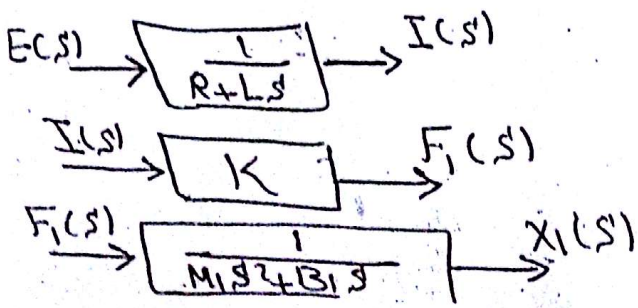


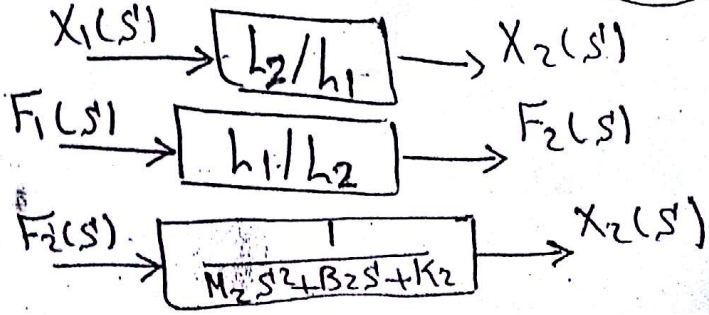
$$\sum F's = M_2 \ddot{X}_2$$

$$F_2 - F_{K_2} - F_{B_2} = M_2 \ddot{X}_2$$

$$F_2 - K_2(X_2 - 0) - B_2(\dot{X}_2 - 0) = M_2 \ddot{X}_2$$

$$\therefore F_2 = M_2 \ddot{X}_2 + B_2 \dot{X}_2 + K_2 X_2 \xrightarrow{L.T} F_2(s) = (M_2 s^2 + B_2 s + K_2) X_2(s)$$





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Block diagram For the system $\frac{X_2(s)}{E(s)}$ (33)

$$\frac{X_2}{E} = \frac{K(L_1/L_2)}{(R+Ls^2)(M_2s^2+B_2s+k_2}$$

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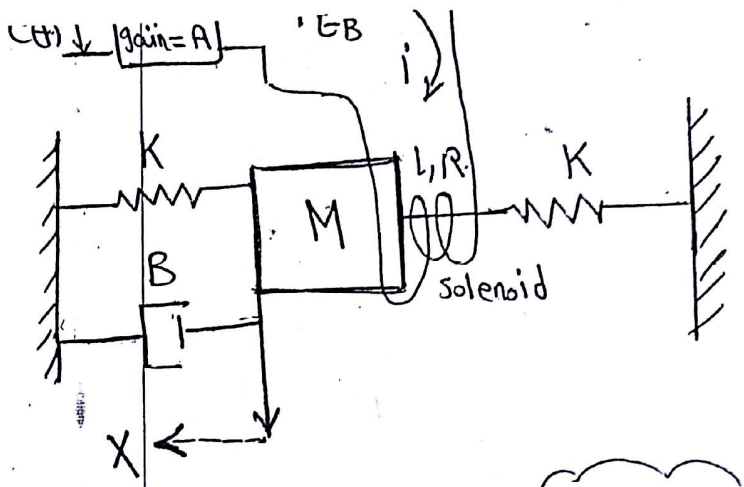
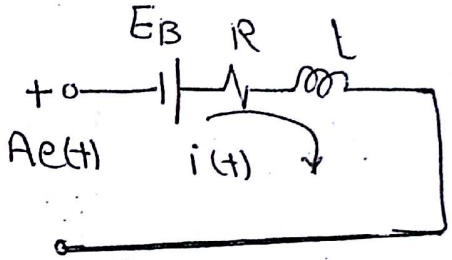


The solenoid produce amgnetic Force $F_c = K_c i$

Find $T.F = \frac{X(s)}{E(s)}$

For electrical circuit



K.V.L ⇒

$$Ae(t) + E_B = iR + L\frac{di}{dt}$$

There are Two i/p's here e(t) & E_B and we want o/p

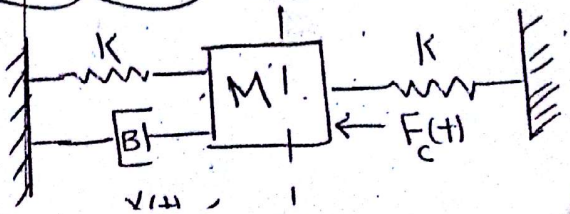
due to e(t) ⇒ (we will assume $E_B = 0$)

$$\Rightarrow Ae(t) = iR + L\frac{di}{dt}$$

L.T ⇒ $AE(s) = (R + Ls)I(s)$

$$\Rightarrow E(s) = \frac{1}{A}(R + Ls)I(s) \quad (1)$$

For Mech. Part



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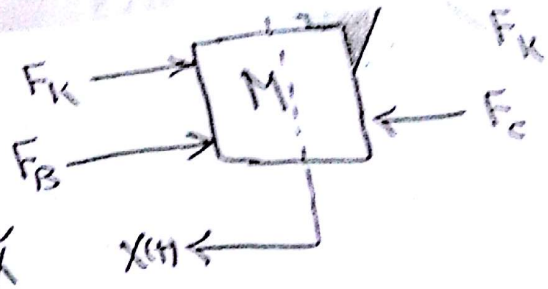

$$\sum F's = M\ddot{x}$$

$$F_c - F_K - F_B - F_{K_1} = M\ddot{x}$$

$$F_c - K(x-0) - B(\dot{x}-0) - K(x-0) = M\ddot{x}$$

$$F_c = M\ddot{x} + B\dot{x} + 2Kx$$

$$\text{L.T} \Rightarrow \boxed{F_c(s) = (Ms^2 + Bs + 2K)X(s)} \quad (2)$$

$$\because F_c = K_c i$$

$$\Rightarrow F_c(s) = K_c I(s) \Rightarrow I(s) = \frac{1}{K_c} F_c(s)$$

$$\therefore \boxed{I(s) = \frac{1}{K_c}(Ms^2 + Bs + 2K)X(s)} \quad (3)$$

From (3) in (1)

$$\Rightarrow E(s) = \frac{1}{A}(R + Ls)\frac{1}{K_c}(Ms^2 + Bs + 2K)X(s)$$

$$T.F = \frac{X(s)}{E(s)} = \frac{}{\frac{1}{AK_c}(R+Ls)(Ms^2+Bs+2K)}$$

$$\therefore \boxed{T.F = \frac{AK_c}{(R+Ls)(Ms^2+Bs+2K)}}$$

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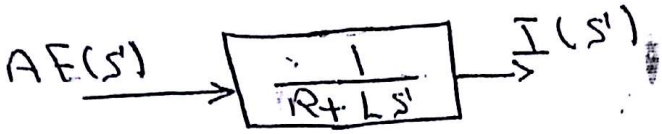
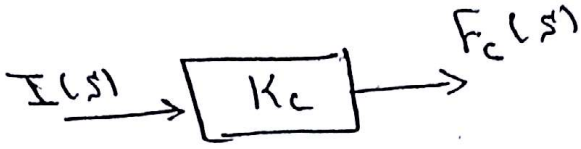
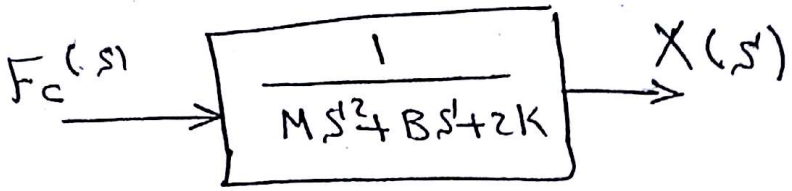
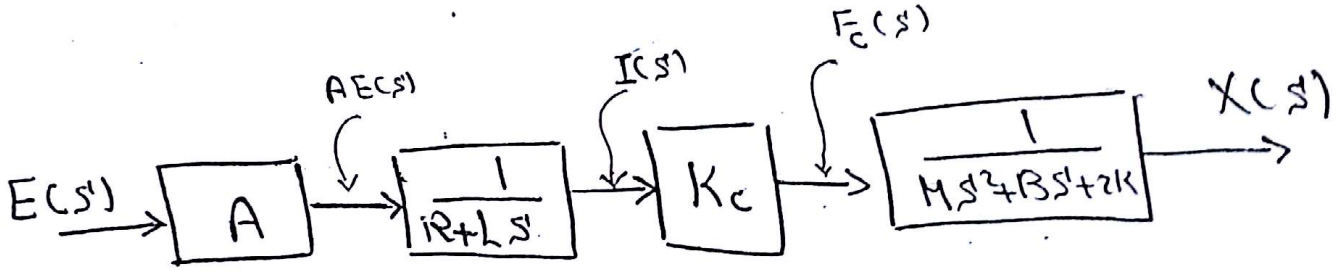
$(m_2 s^2 + K) X_2(s) + K X_1(s) = 0$

e T.F of each Block



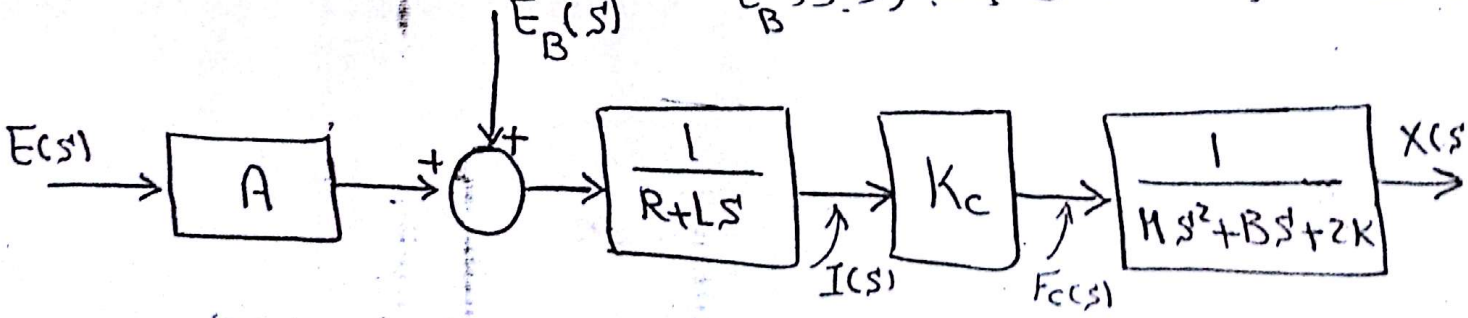






Block diagram incase of only one i/p (E(s))

* في حالة إذا أخذنا في الإعتبار وجود E_B



Block diagram in case of two i/p E_B & E(s)

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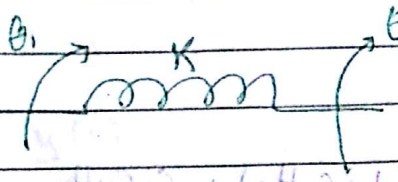
الحركة الخطية	الحركة الدائرية
$X(t)$	$\theta(t)$
كتلة M	J inertia العزم الذاتي للجسم
قوة $F(t)$	عزم $T(t)$

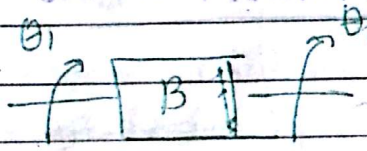
$\sum F's = Mx''$ ← لحركة خطية

$\sum T's = J\theta''$ ← لحركة دائرية



$T_K = K(\theta_1 - \theta_2)$

عزم دوران الفعل
في الـ spring



$T_B = B(\theta_1' - \theta_2')$

عزم دوران الفعل من الـ damper.

قانون نيوتن الثاني للحركة الدائرية

$$\sum T's = J\theta''$$

عزم دوران أقياه العزم ← input Torque $T(t)$

عزم دوران الفعل ← T_K أو T_B

$$T(s) = (J_1 s^2 + B_1 s + K)\left(\frac{1}{K}(J_2 s^2 + B_2 s + K)\right)\theta_2 - K\theta_2$$

$$\frac{T}{F} = \frac{K}{(J_1 s^2 + B_1 s + K)(J_2 s^2 + B_2 s + K) - K^2}$$

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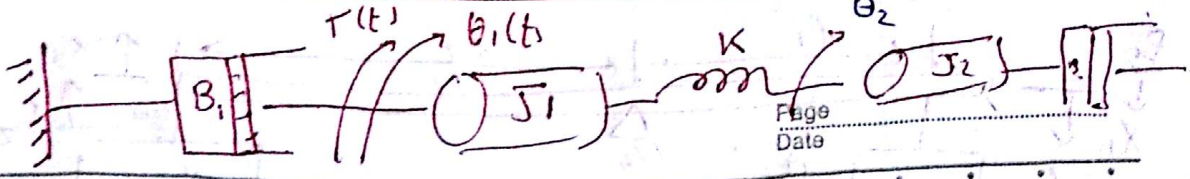

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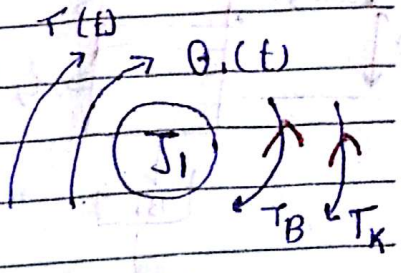
Date

Find T.F. $\frac{\theta_2(s)}{T(s)}$



at J_1:

$\sum T's = J\theta_1''$

$T(t) - T_B(t) - T_K(t) = J\theta_1''$

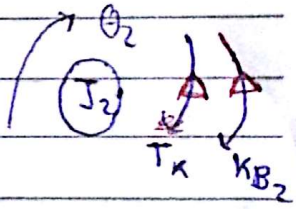
$T(t) - B_1(\dot{\theta}_1 - 0) - K(\theta_1 - \theta_2) = J\theta_1''$

L.T. ↓

$T(s) - sB_1\theta_1 - K\theta_1 + K\theta_2 = Js^2\theta_1$

$T(s) = (J_1s^2 + B_1s + K)\theta_1 - K\theta_2 \Rightarrow (1)$

at J_2:



$\sum T's = J_2\theta_2''$

$0 - T_K - T_{B_2} = J_2\theta_2''$

$-K(\theta_2 - \theta_1) - B_2(\dot{\theta}_2 - 0) = J_2\theta_2''$

↓ L.T.

$-K\theta_2 + K\theta_1 - B_2s\theta_2 = J_2s^2\theta_2$

$K\theta_1 = (J_2s^2 + B_2s + K)\theta_2$

$\theta_1 = \frac{1}{K}(J_2s^2 + B_2s + K)\theta_2 \Rightarrow (2)$

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